APPLICATION OF LEARNING FEED-FORWARD CONTROLLER BASED ON MODEL REFERENCE ADAPTIVE SYSTEM FOR MISSILE STABILIZATION


Abstract. The article presents the results of research, analysis and how to build learning feed-forward controller based on model reference adaptive system in the remote control loop for missile stabilization. The controller structure is simple, adaptive control law applying Lyapunov stability theory fast convergence and sustainable. The simulation results have shown the advantages of using algorithm, the missile is always stable when there is a parameter change due to the effects of flight conditions.

Keywords: Learning feed-forward controller, Model reference adaptive system, Missile, Stabilize.

1. INTRODUCTION

The nature of applying an adaptive control system according to a reference model is to design the controller so that the system to achieve the desired properties given by a mathematical model (reference model) [2]. When the properties of the real system different from the ideal properties of the reference model, the system is changed by adjusting the parameters of the controller or create additional sub-signal [1].

The learning feed-forward controller is based on a model reference adaptive system capable of automatically adjusting the parameters of the controller according to the tendency to bring error (ε) between the reference model and the process (missile) forward gradually go to zero. The advantages of this controller are fast adaptive speed, high stability and less sensitivity to noise [1, 2].

Missiles in the remote control loop is kinetic stage with change parameters, and therefore need to be stabilized. The change in the missile normal accelerations depends on the wing deflection angle in groove nod described by the transmission function [3, 4, 5, 6]:

$$K_p(p) = \frac{k_p v_p}{\tau_p^2 + 2\xi_p \tau_p p + 1}$$  \hspace{1cm} (1)

There have been a number of articles providing solutions to stabilize the missile by application of adaptive control theory [1, 2]. However, the solutions proposed today mainly use the feedback linearization method, complex stability algorithm, which requires many measuring set (or evaluation) of the missile’s kinematic parameters [1, 4]. Therefore, the realization of the algorithm is very difficult.

Seeing that, to improve the accuracy of destroying the target, the missile needs to be stable during flight. Therefore, the article proposes how to stabilize the missile on the application of learning feed-forward controller based on model reference adaptive system. The controller has a simple structure, the law adapted fast convergence and sustainable. Algorithm is verified through simulation, has reliable results and is able to realize the algorithm in current technological and technical conditions.

2. DESIGN ADAPTIVE LEARNING FEED-FORWARD CONTROLLER ACCORDING TO THE REFERENCE MODEL

The structure depicted in figure 1 can be used as an model reference adaptive system [1, 2]. The process has a mathematical model of the second order, which is controlled with the help of the learning feed-forward controller. The parameters of this controller are $a_m, b_m, c_m$. The change parameter of the process

Inside; $K_p$ - Transfer coefficient of the missile

$v_p$ - Missile velocity

$\tau_p$ - Time constant

$\xi_p$ - The attenuation coefficient fluctuates individually

$\tau_V$ - Aerodynamic time constant

$\tau_V$ depends on the aerodynamic arrangement of the missile, the geometrical and aerodynamic characteristics of the missile’s elements. They change with flight conditions (altitude, velocity, change of attack angle ...) [5, 6]. In particular, the $\frac{v_p}{\tau_V}$ coefficient varies greatly, depending on the dynamic pressure and make amplification coefficient of the control system also change within a wide limit.

There have been a number of articles providing solutions to stabilize the missile by application of classical control theory [5]. However, only response within a certain range the dependence of the open control loop gain on the aerodynamic pressure [5]. Stabilizing the missile stage parameters requires complex equipment, multiple sensors, each parameter needs a separate stabilizer [4, 6]. Although stable solutions have been implemented, but in actual the missile stage parameters still change, so the quality of the control loop will decrease, the missile stage parameter is different from the calculated parameters [5, 6].
(missile) is \( a, b, c \). Normal acceleration requires of the missile stage input is \( a_r \) (control signal).

The reference model is described by:

\[
\begin{align*}
\frac{y_m}{a_r} &= \frac{\omega_m^2}{p^2 + 2\xi_m \omega_m p + \omega_m^2} \\
\end{align*}
\]  

(2)

The process is described by:

\[
\begin{align*}
\dot{x} &= \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{a}{c} & -\frac{b}{c} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u = Ax + Bu \\
\end{align*}
\]  

(4)

By means of a learning feed-forward controller, the state variable filter output signals can be used to create an inverse model of the process. We need to define the operating principle based on the errors between the output of the reference model \( y_m \) and the output of the process \( x \), and adjust the parameters \( a_m, b_m \) and \( c_m \) so that they converge according to the parameters of process \( a, b \) and \( c \) respectively.

This shows that we can use the approach by classic Lyapunov stability theory to find the law of adaptation for the learning feed-forward controller parameters.

- Step 1: Determine the differential equation for \( e \)

Represent the reference model as a state variable:

\[
\begin{align*}
\dot{y}_m &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} y_m \\ y_{m2} \end{bmatrix} + \begin{bmatrix} 0 & 1 \end{bmatrix} \epsilon = \begin{bmatrix} 0 & 1 \end{bmatrix} y_m + \begin{bmatrix} 0 & 1 \end{bmatrix} \epsilon \\
\end{align*}
\]  

(5)

Therefore, the design problems posed are: Find (stabilize) the adjustment law for the adjustment parameters \( a_m, b_m \) and \( c_m \) so that the error between the reference model \( e \) and the process progresses to 0, and adjust the parameters \( a, b_m \) and \( c_m \) so that they converge according to the parameters of process \( a, b \) and \( c \) respectively. Steps to design the controller adapted with Lyapunov stability theory as follows:
Represents a process as a state variable:

\[ \dot{x}_1 = x_2 \]  

(7)

\[ \dot{x}_2 = -\frac{c}{a} x_1 - \frac{b}{a} x_2 + \frac{1}{a} (c_m y_{m1} + b_m y_{m2}) + \frac{1}{a} a_m \epsilon \]  

(8)

\[ \dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -\frac{c}{a} & 1 \\ -\frac{b}{a} & \frac{1}{a} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{c_m}{a} \\ \frac{b_m}{a} \end{bmatrix} y_{m1} + \begin{bmatrix} \frac{1}{a} a_m \end{bmatrix} \epsilon 
\]  

(9)

With error \( e \) is determined by the following formula:

\[ e = y_m - x \]  

(10)

\[ \dot{e} = \dot{y}_m - \dot{x} \]  

(11)

Replace (6) and (9) in to (11) we have:

\[ \dot{e} = \begin{bmatrix} \frac{c_m}{a} & 0 \\ \frac{b_m}{a} & \frac{1}{a} \end{bmatrix} y_m - \begin{bmatrix} -\frac{c}{a} & 1 \\ -\frac{b}{a} & \frac{1}{a} \end{bmatrix} x + \begin{bmatrix} \frac{1}{a} a_m \end{bmatrix} \epsilon \]  

(12)

\[ \begin{align*}
A_1 &= \begin{bmatrix} \frac{c}{a} & \frac{b_m}{a} \\ -\frac{c}{a} & \frac{1}{a} \end{bmatrix},
A_2 &= \begin{bmatrix} -\frac{c}{a} & \frac{1}{a} \\ \frac{b}{a} & \frac{1}{a} \end{bmatrix},
B_1 &= \begin{bmatrix} 0 & 0 \\ 1 & a_m \end{bmatrix}
\end{align*} \]  

(13)

- Step 2: Select the Lyapunov \( V(e) \) function [1, 2]

\[ V(e) = e^T N e + a^T a \epsilon + b^T \beta \epsilon \]  

(14)

Inside, \( N \) - Symmetric matrix is determined arbitrarily positive.
\( a \) and \( b \) - The vectors contain nonzero elements of matrices \( A_1 \) and \( B_1 \).
\( \alpha \) and \( \beta \) - The diagonal matrix contains the elements that determine the adaptation process speed.

- Step 3: Determine the conditions for function \( \dot{V}(e) \) to determine negative

\[ \dot{V}(e) = (A_1 y_m + A_2 e + B_1 \epsilon)^T N e + e^T N (A_1 y_m + A_2 e + B_1 \epsilon) + 2 \dot{a} \dot{a}^T + 2 \dot{b} \dot{b}^T \]  

(15)

From [1, 2]: \( A^T N + NA = -Q \) either \( e^T (A^T N + NA) e = -e^T Q e \)

According to Malkin’s theorem, \( Q \) is a positive deterministic matrix. This means that the value for part

\[ e^T N A_1 y_m + \dot{a} \dot{a}^T = 0 \]  

(16)

\[ e^T N B_1 e + \dot{b} \dot{b}^T = 0 \]  

(17)

With \( a = \begin{bmatrix} a_{21} & a_{22} \end{bmatrix}, e = \begin{bmatrix} e_1 & e_2 \end{bmatrix}, \alpha = \begin{bmatrix} 0 & 0 \\ a_{21} & a_{22} \end{bmatrix}, N = \begin{bmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{bmatrix}, A_1 = \begin{bmatrix} 0 & 0 \\ a_{21} & a_{22} \end{bmatrix}, y_m = \begin{bmatrix} y_{m1} \\ y_{m2} \end{bmatrix} \)

After calculating, we get the following results:

\[ \dot{a}_{21} = -\frac{1}{a_{21}} (e_1 n_{21} + e_2 n_{22}) y_{m1} \]  

(18)

\[ \dot{a}_{22} = -\frac{1}{a_{22}} (e_1 n_{21} + e_2 n_{22}) y_{m2} \]  

(19)

From formula (9) we have:

\[ a_{21} = \frac{c}{a} - \frac{c_m}{a} \rightarrow \dot{a}_{21} = -\frac{1}{a} c_m \]  

(20)
To complete the parameter update process, \( c_m \) is defined by the following expression:

\[
c_m = \frac{a}{a_{13}} \int [(e_1, n_{21} + e_2 n_{22}) y_{m1}] \, dt + c_m(0)
\]  

(21)

From formula (13) we have:

\[
a_{22} = \frac{b}{a} - \frac{b_m}{a} \rightarrow \tilde{a}_{22} = -\frac{1}{a} b_m
\]  

(22)

To complete the parameter update process \( b_m, a_m \) are defined by the following expression:

\[
b_m = \frac{a}{a_{22}} \int [(e_1, n_{21} + e_2 n_{22}) y_{m2}] \, dt + b_m(0)
\]  

(23)

\[
a_m = \frac{a}{a_{22}} \int [(e_1, n_{21} + e_2 n_{22}) \varepsilon] \, dt + a_m(0)
\]  

(24)

Where \( a_{22} \) and \( b_{22} \) are called the speed of the adaptation process, \( n_{21} \) and \( n_{22} \) are the elements of the matrix \( N \).

- **Step 4**: Determine \( N \) from \( A^T N + NA = -Q \) with \( Q = \begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix} \)

\[
\begin{bmatrix} 0 & -\frac{c}{a} \\ 1 & -\frac{b}{a} \end{bmatrix} \begin{bmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{bmatrix} + \begin{bmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -\frac{c}{a} \frac{b}{a} \end{bmatrix} = -\begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix}
\]

(25)

\[
\begin{bmatrix} -\frac{c}{a} (n_{21} + n_{22}) & n_{11} - \frac{c}{a} n_{22} - \frac{b}{a} n_{12} \\ n_{11} - \frac{b}{a} n_{21} - \frac{c}{a} n_{22} & n_{12} + n_{21} - 2 \frac{c}{a} n_{22} \end{bmatrix} = -\begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix}
\]

(26)

The results are as follows:

\[
n_{12} = n_{21} = \frac{1}{2} q_{11} - \frac{a}{c}
\]

(27)

\[
n_{11} = n_{22} = \frac{1}{2} (\frac{a^2}{c^2} q_{11} + \frac{a}{c} q_{22})
\]

(28)

From formula (21), (23) and (24), the design of the learning feed-forward controller based on model reference adaptive system with Lyapunov’s stability theory in figure 1 is redrawn in figure 2 as follows:

![Figure 2. The learning feed-forward controller based on model reference adaptive system](attachment:image.png)
3. SIMULATION RESULTS AND ANALYSIS

The algorithm survey is performed within the remote missile control loop [5, 6]. The simulation organization diagram is in the form of figure 3, with the parameters are selected follows:

- Target has speed $V_t = 450 (m/s)$, flying in, the horizontal distance $D = 29 (km)$, altitude $H = 6 (km)$, maneuver start time at moment $t_1 = 10 (s)$, maneuver finish at moment $t_2 = 15 (s)$, maneuver $30 (m/s^2)$.
- Missile velocity $V_p = 900 (m/s)$.
- Target coordinate determination system
- Command setting system
- The missile has adaptive system
- Missile coordinate determination system
- Creating fake targets

$$a = 0.011; b = 0.15; c = 1.$$  
- Controller parameters: $\omega_m = 10; \xi_m = 0.7$.  
- $Q = \begin{bmatrix} 7 & 14 \\ 14 & 7 \end{bmatrix}$ then $p_{21} \approx 0.0385; p_{22} \approx 0.078$.
- The command setting system creates command according to the 3-point guidance method.

Figure 3. Structure diagram of control loop using learning feed-forward controller based on model reference adaptive system

Figure 4. Missile trajectory – target

Figure 5. Error at meeting point
**Comment:** The missile is always stable during flight. The controller is adaptable to changes in kinetic parameters, the error at the meeting point (straight deviation) is small. Error between the reference model and missile is small, the law of adaptation is fast convergence and sustainable.

4. **CONCLUSION**

The controller structure is simple, adaptive control law according to the reference model applied Lyapunov stability theory fast convergence and sustainable. Simulation results show that, when using an adaptive mechanism, it is more stable than when not in use. This is the basis of improving the accuracy destroy targets, meet in the actual conditions when the missile’s flight conditions change.

However, the adaptive laws (21), (23) and (24) only apply to the process and the reference model has the second order transfer function. Thus, when the process has a higher order transfer function, we must approximate that transfer function to degree order 2. The learning feed-forward controller only compensates and corrects for the process with the second order transfer function, which is a limitation of this method.

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